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Lattice Boltzmann simulations of acoustic streaming

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Abstract
Lattice Boltzmann simulations are used to model the acoustic streaming produced by the interaction of an acoustic wave with a boundary. Comparison is made with analytic results for Rayleigh streaming in the appropriate limits and we show how deviations from those limits affect the streaming. Acoustic streaming patterns around a cylinder and between two plates of finite length are then predicted.

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1. Introduction

Acoustic streaming describes a steady flow field superimposed upon the oscillatory motion of a sound wave propagating in a fluid. It is a nonlinear effect which occurs due to the presence of boundaries or because of damping of the wave.

Acoustic streaming velocities are small compared with the oscillatory fluid motion ($\sim$0.1%) However, there are reports in the literature that the streaming can enhance rate-limited processes such as diffusion [1,2], heat transfer [3] and the rate of sonoelectrochemical reactions [4]. The reasons behind this behaviour are unclear and to understand and control the enhancement it is important to better characterize the flow fields produced by acoustic streaming. For a few simple systems acoustic streaming velocities have been determined analytically. However, for more complex systems, analytical solutions are likely to prove too difficult, so modelling methodologies need to be found. The aim of this paper is to describe one such approach, lattice Boltzmann simulations, which predicts acoustic streaming patterns from a direct solution of the Navier–Stokes equations.

There are two basic types of acoustic streaming. The first, which has received the majority of attention to date, is Rayleigh or Schlichting streaming. This is caused by relative oscillatory motion between the fluid and a boundary. The steady flow results from the rapid change in the wave amplitude in the acoustic boundary layer. The effects of attenuation are usually considered to be negligible. The second, Eckart streaming, results from the attenuation of the wave in the bulk fluid. Here the momentum transfer from the wave is converted into a time-averaged flow moving away from the source. This is commonly referred to as a 'quartz wind' because it was first discovered with the introduction of high-amplitude quartz transducers.

Acoustic streaming was first considered analytically by Rayleigh in 1884 [5]. He predicted the streaming pattern for a standing wave between two parallel plates. More recently Nybourg [3] corrected this solution and presented results for the streaming caused by a travelling wave between plates and for streaming around an infinitely long cylinder.

To date there have been two studies that have modelled acoustic streaming. Nightingale *et al* [6] used a standard computational fluid dynamics package to predict the flow produced in breast cysts by Eckart streaming. The authors used the analogy between streaming and slow viscous flow [3] to model the streaming as the flow produced by a body force which was calculated from the time-dependent (first-order) wave. This requires the wave profile to be known throughout the area of interest and assumes that there are no effects in going from a stationary to an oscillatory reference frame when performing the streaming calculations.

Stansell *et al* [7] used a lattice gas approach to model the acoustic streaming produced by a standing wave between two parallel plates. This is a simulation of the full Navier–Stokes equations of motion and the acoustic streaming appears as a small correction to the oscillatory flow field. However, lattice gas simulations are inherently very noisy and streaming could only be seen for high amplitudes and viscosities. Whilst the results produced similar velocity profiles to the theoretical predictions there were some noticeable differences, which were attributed to higher-order contributions. This could not be verified as no theory was available which included the higher-order terms.

Lattice Boltzmann simulations were developed from the lattice gas approach as an attempt to overcome noise problems. They can be regarded either as a 'mean-field' lattice gas or as a slightly unusual finite-difference solver for the Navier–Stokes equations [8]. Like the lattice gas, the lattice Boltzmann approach can be used to simulate streaming without having to calculate the forces produced by the wave. Once a sound wave is generated in the model streaming should follow automatically.

Here we demonstrate that this is indeed the case and explore the extent to which lattice Boltzmann simulations can be used to model acoustic streaming. We show that because there is no noise present in the lattice Boltzmann model it is possible to see streaming at amplitudes and viscosities which approach the limits considered in analytic work. This both provides a useful check on the simulations and shows how acoustic streaming behaves beyond the limits amenable to analytic calculation.

The next section of the paper introduces the lattice Boltzmann approach and gives details of the numerical calculations. In section 3 we compare the simulation results for streaming between two infinite parallel plates with analytical results and show how the streaming changes as the assumptions inherent in the theory are violated. Section 4 presents the results for streaming around a cylinder and between plates of finite length.

2. Lattice Boltzmann model

2.1. The algorithm

The lattice Boltzmann model we use is a single-relaxation-time BGK scheme on a two-dimensional, hexagonal lattice [8]. Lattice vectors are $e_i = \cos\{\pi(i-1)/3\},\ i = 1, 2 \ldots 6$, together with a zero velocity vector $e_0 = (0, 0)$. A set of partial densities $f_i(\boldsymbol{x}, t)$ associated with each lattice direction i is defined on each lattice site $\boldsymbol{x}$. They evolve with time t according to

$$f_i(\boldsymbol{x} + \boldsymbol{e}_i \Delta t, t + \Delta t) - f_i(\boldsymbol{x}, t) = -\frac{1}{\tau}(f_i - f_i^0) \tag{1}$$

where the local equilibrium distribution

$$f_i^0 = \rho\{\tfrac{1}{12} + \tfrac{1}{3}\boldsymbol{e}_i \cdot \boldsymbol{u} + \tfrac{2}{3}(\boldsymbol{e}_i \cdot \boldsymbol{u})^2 - \tfrac{1}{6}(\boldsymbol{u})^2\} \qquad i = 1, 2 \ldots 6 \tag{2}$$

$$f_0^0 = \rho\{\tfrac{1}{2} - (\boldsymbol{u})^2\} \tag{3}$$

and $\rho(\vec{x}, t)$ and $\boldsymbol{u}(\vec{x}, t)$ are the density and velocity respectively. They are related to the partial densities by

$$\sum_i f_i(\boldsymbol{x}, t) = \rho(\boldsymbol{x}, t) \qquad \sum_i f_i(\boldsymbol{x}, t)\boldsymbol{e}_i = \rho(\boldsymbol{x}, t)\boldsymbol{u}(\boldsymbol{x}, t). \tag{4}$$

In equation (1) the left-hand side represents the propagation step, where momentum is transferred between lattice points, and the right-hand side represents the relaxation process which determines the viscous properties of the fluid. In the continuum limit a Chapman–Enskog expansion shows that the numerical scheme reproduces the Navier–Stokes equation [8]

$$\partial_t \rho + \partial_\alpha \rho u_\alpha = 0 \tag{5}$$

$$\partial_t \rho u_\beta + u_\beta \partial_\alpha \rho u_\alpha + \rho u_\alpha \partial_\alpha u_\beta = -\partial_\alpha P + \rho\nu\partial_\alpha[\partial_\beta u_\alpha + \partial_\alpha u_\beta] \tag{6}$$

with pressure $P = \rho/4$, kinematic viscosity $\nu = (2\tau - 1)/8$ and speed of sound $c_0 = 1/2$.

2.2. *Choice of parameters*

Provided the same values for ∇P and ν are used the absolute value of the density or pressure in the lattice Boltzmann model or the Navier–Stokes equations has no effect on the flow field produced. For convenience an undisturbed density $\rho_0 = 1$ was used. The relaxation time for all the simulations was $\tau = 0.525$ ($\nu = 0.006\,25$). This represented the best compromise between reducing the attenuation of the wave in the model, which is a function of viscosity, and producing a simulation that contains no numerical oscillation at the second order.

For the model to simulate streaming in a channel appropriate boundary conditions must be imposed. These are periodic boundary conditions along the channel and non-slip conditions at the sides of the channel. The non-slip conditions are produced by a standard bounce-back scheme where mass leaving the system remains at the same site but with its velocity reversed [9].

We considered a channel of length L_x in the direction of propagation of the sound wave and width L_y (corresponding to $2L_y/\sqrt{3}$ lattice points) and imposed a sound wave of wavelength $\lambda = L_x$ for all the simulations. The theoretical results for streaming between two parallel plates are obtained in the limits $L_x, \lambda \gg L_y$ and $L_y \gg \beta^{-1}$ where β^{-1} is the width of the acoustic boundary layer. Moreover attenuation of the standing wave is assumed to be negligible. Satisfying all these conditions simultaneously presents a problem for the simulation.

For a long thin channel the boundary conditions at the wall significantly increase the attenuation. A model with a high aspect ratio and low attenuation will require a very large lattice, typically $L_x \sim 8000$, $L_y \sim 692.8$, which will be computationally expensive.

To reduce the attenuation it is necessary to use a channel width that is much closer to the wavelength or possibly wider. Therefore as a compromise four lattice sizes have been considered: two ($L_x = 1000$, $L_y = 86.6$; $L_x = 200$, $L_y = 86.6$) to investigate the effect of the aspect ratio of the lattice and two ($L_x = 540$, $L_y = 433$; $L_x = 270$, $L_y = 138.6$) to simulate waves with a relatively low attenuation. The 540 by 433 lattice model took 24 h to run on a single-processor R10000 workstation.

2.3. The first-order standing wave

Acoustic streaming occurs at second order in the wave amplitude. In the simulations presented here the first-order velocity of the sound wave is of the order of 1000 times the streaming velocity. One of the consequences of this is that a small change in first-order wave will have a profound effect on the values obtained for the streaming. It is therefore essential that the wave is maintained in a stable condition throughout the simulation.

The theory of acoustic streaming assumes the first-order sound wave has sufficient amplitude to produce the nonlinear effect of streaming, whilst other nonlinear effects, such as the generation of higher harmonics and shock fronts, can be ignored. However, when waves of sufficient amplitudes to generate significant streaming velocities propagate in lattice Boltzmann models nonlinear propagation effects will be important [10]. This will lead to the sound wave taking on a saw-tooth form, which contains a significant number of harmonics which are not accurately represented by the theory. To avoid this the wave was maintained by imposing a sinusoidal pressure variation at $L_x = 0$ by a suitable choice of the partial distribution functions at each time step. This ensured the wave remained linear and at the correct amplitude because $x = 0$ effectively acted as the source of the wave and did not allow any reflections of the returning wave.

3. Analytical results for streaming between parallel plates

Consider a standing wave of wavevector $k = 2\pi/\lambda$ and frequency ω in a channel of infinite length along the x-direction and width L_y along the y-direction. The Navier–Stokes equations can be solved to first order in the wave velocity $\boldsymbol{u_1}$ to give a standing wave solution [3] for the x and y components of velocity

$$\begin{aligned} u_{1x} &= U_1 \sin kx\{\sin(\omega t) - \mathrm{e}^{-\beta y}\sin(\omega t - \beta y)\} \\ u_{1y} &= \frac{U_1 k}{\sqrt{2}\beta}\cos kx\left\{\sin\left(\omega t - \frac{\pi}{4}\right) - \mathrm{e}^{-\beta y}\sin\left(\omega t - \beta y - \frac{\pi}{4}\right)\right\} \qquad 0 \leqslant y \leqslant L_y/2 \end{aligned} \tag{7}$$

where $\beta^{-1} = \sqrt{2\nu/\omega}$ is the width of the acoustic boundary layer, U_1 is the magnitude of the velocity amplitude as $\beta \to \infty$ and the solution for $L_y/2 < y < L_y$ follows by symmetry.

These expressions are derived under the assumptions that $\beta^{-1} \ll L_y, \lambda$. Moreover it is assumed that the derivatives of the velocity with respect to x are negligible compared with the y-derivatives. This is a good assumption within the boundary layer. However, it leads to the non-physical result that $u_{1y} \neq 0$ at the centre of the channel. This is unlikely to affect the streaming significantly as $u_{1y} \ll u_{1x}$.

Equations (7) can be used to obtain the second-order terms in the amplitude expansion. The time-averaged components of these for the net mass flow are the streaming velocity as seen by a tracer particle [3]

$$u_{2x} = \frac{3U_1^2}{8c}\sin 2kx\left\{\mathrm{e}^{-2\beta y} + 2\mathrm{e}^{-\beta y}\sin\beta y + 6\frac{y}{L_y}\left(1 - \frac{y}{L_y}\right) - 1 + \frac{18y}{\beta L_y^2}\left(\frac{y}{L_y} - 1\right)\right\} \tag{8}$$

$$\begin{aligned} u_{2y} = \frac{3U_1^2 k}{8\beta c}\cos 2kx \Bigg\{ &\mathrm{e}^{-2\beta y} + \mathrm{e}^{-\beta y}(\sin\beta y + \cos\beta y) \\ &+2\beta\left(1 - \frac{2y}{L_y}\right)\left(1 - \frac{y}{L_y}\right) + 6\left(\frac{y}{L_y}\right)^2\left(3 - \frac{2y}{L_y}\right) - 3\Bigg\} \end{aligned} \tag{9}$$

where c is the speed of sound in the fluid.

4. Comparison of theory and simulation

4.1. First-order wave

We first compare the form of the standing wave set up in the channel with the theoretical predictions (7). Results are shown for a lattice of size $L_x = 200$, $L_y = 86.6$. The wave was initialized as $\rho = \rho_0 + \triangle\rho_1 \cos(kx)$, $\boldsymbol{u} = 0$ with $\triangle\rho_1 = 0.01$.

Figures 1(*a*) and (*b*) show the variation of u_{1x} and u_{1y} with y at their velocity antinodes. In the centre of the channel the simulated velocity u_{1x} is lower than the calculated value by $\sim$2%. This is likely to be due to attenuation of the wave ($\sim$3% over one cycle, primarily due to the boundaries), which will lead to a reduction in velocities at the antinodes. To check this

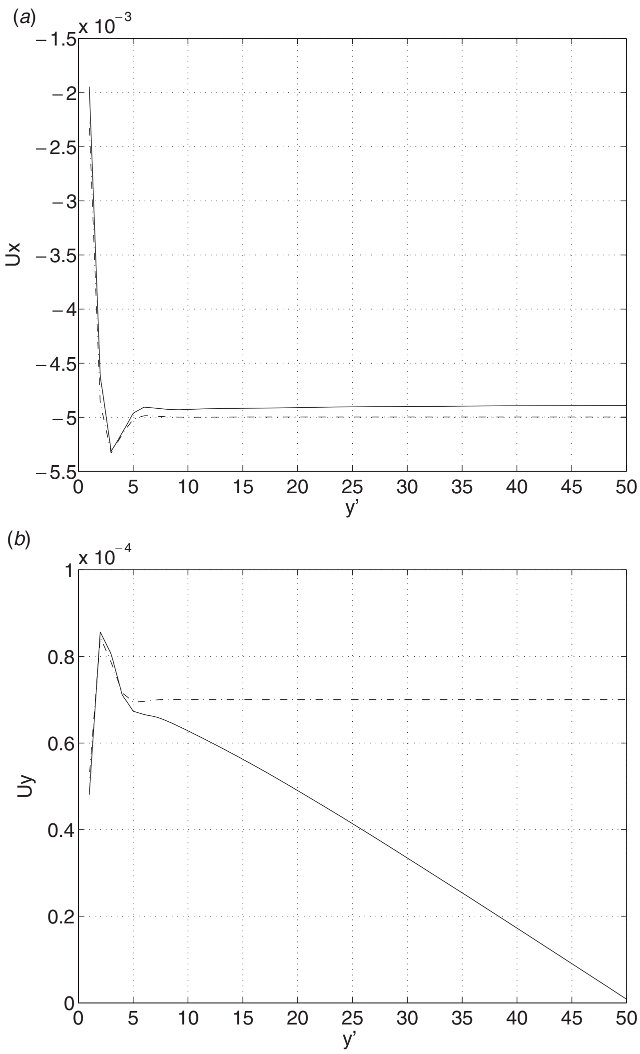


Figure 1. First-order wave velocity at the velocity antinode. (*a*) x component; (*b*) y component ($L_x = 200$, $L_y = 86.6$, $\tau = 0.525$, $\triangle\rho = 0.01$, $y' = 2y/\sqrt{3}$). The solid curve is the simulation; the theory is the dashed curve.

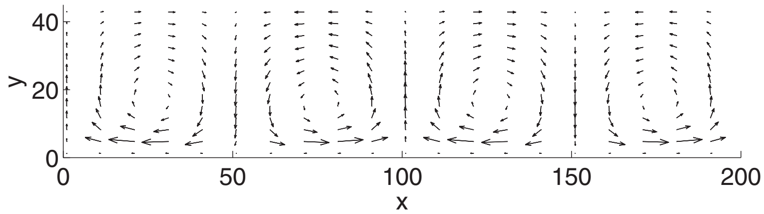


Figure 2. Acoustic streaming velocity vector plot ($L_x = 200$, $L_y = 86.6$, $\tau = 0.525$, $\triangle\rho = 0.01$).

we obtained similar results for a lattice of length $L_x = 1000$. Here the attenuation is $\sim$7% and the discrepancy between theory and simulation increased to $\sim$5%. The variation of the velocity through the boundary layer is reproduced well by the simulations.

The simulated and calculated values for u_{1y} are similar in the boundary layer. Moving towards the centre away from the boundaries they diverge because the simulations give the correct value $u_{1y} = 0$ at the centre of the channel. Note, however, that the magnitude of u_{1y} is small everywhere and it is unlikely to significantly affect the streaming.

4.2. Streaming

Acoustic streaming is the local velocity averaged over one period of the sound wave. However, to increase the sensitivity in all of the following simulations the streaming velocity has been taken as the average velocity over 50 cycles, after an initialization period of 200 cycles to allow streaming to reach a steady state.

Figure 2 shows the velocity field for acoustic streaming for a channel of width $L_y = 86.6$ and $L_x = 200$. As expected there are two series of vortices with period $\lambda/2$ lying symmetrically about the centre of the channel. There is a local maximum close to the $y = 0$ boundary due to the high first-order velocity gradient in this area. The ratio of streaming velocities to the oscillatory velocity of the standing waves $\sim 2 \times 10^{-3}$.

Details of the velocity field are compared with the theoretical predictions for three different lattice sizes ($L_x = 1000$, $L_y = 86.6$; $L_x = 200$, $L_y = 86.6$, and $L_x = 560$, $L_y = 433$) in figures 3, 4 and 5 respectively. In each case the variation of u_{2x} and u_{2y} with y is plotted at a velocity antinode (the maximum velocity, which occurs at $x = \lambda/4$ for u_{2y} and $x = \lambda/8$ for u_{2x}). We stress that exact agreement cannot be expected because the theoretical assumptions are not exactly reproduced in the model. However, by comparing different lattice sizes we can check that the right behaviour is seen as the simulations approach the theoretical limits and present new results showing how the streaming behaves for parameter values inaccessible to the theory.

Consider first ($L_x = 1000$, $L_y = 86.6$). This aspect ratio is the closest to the theoretical limit $\lambda \gg L_x$ and the numerical results lie close to the theoretical curves. Away from the boundary the model gives a slightly lower value than the theory. This is likely to be due to the attenuation ($\sim$7% per cycle) which reduces the amplitude of the standing wave. One of the theoretical assumptions is that x-derivatives are negligible compared with y-derivatives. For this aspect ratio we measure $\partial u_1/\partial x \sim 1\%\ \partial u_1/\partial y$ and $\partial u_2/\partial x \sim 5\%\partial u_2/\partial y$.

The simulations reproduce the streaming velocity in the boundary layer surprisingly well given the large changes over a small number of lattice sites. The build-up of the velocity happens slightly closer to the boundary in the simulations. At first sight this looks like an inaccuracy caused by errors in the bounce-back non-slip boundary conditions. However, when the results were compared with those produced by a scheme that ensures zero velocity

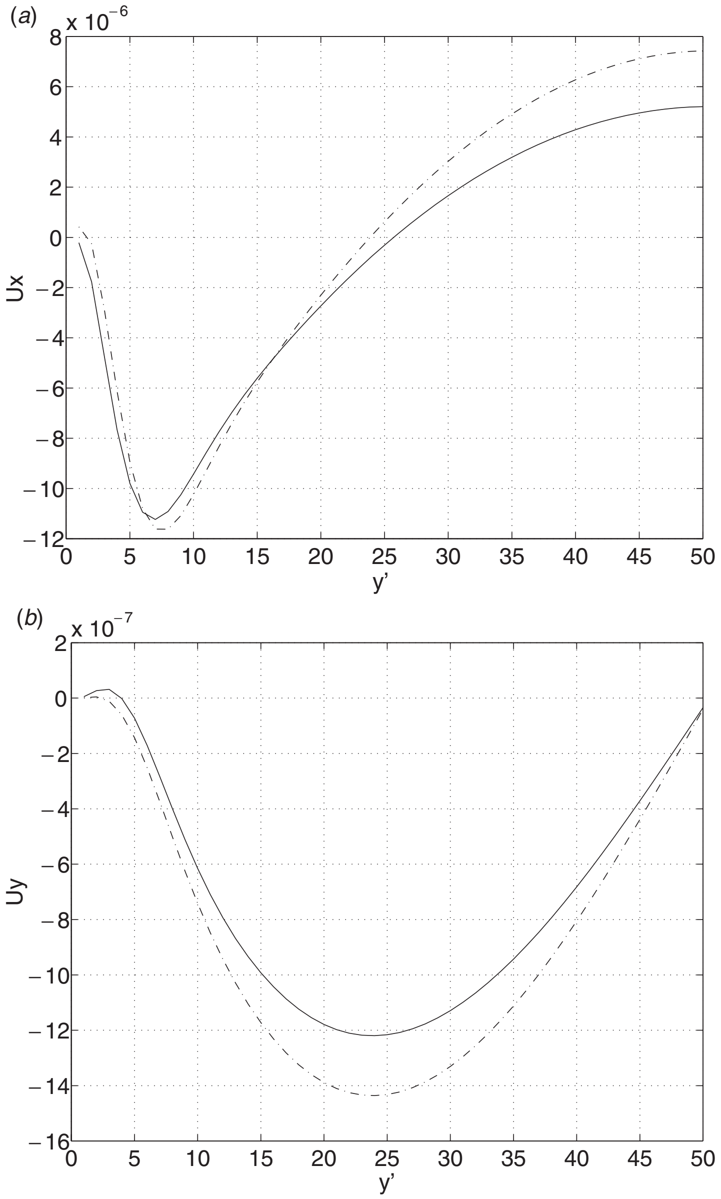


Figure 3. Acoustic streaming at the velocity antinode: (*a*) x-component; (*b*) y-component ($L_x = 1000$, $L_y = 86.6$, $\tau = 0.525$, $\triangle\rho = 0.01$, $y' = 2y/\sqrt{3}$). The solid curve is the simulation; the theory is the dashed curve.

exactly at the boundary [11] there was no difference in the accuracy. Close examination of the theoretical prediction shows a small reversal in the direction of the streaming velocity close to the boundary. This change is not present in the simulation presented here and is the most likely explanation of the displacement of the simulation velocity from the theoretical velocity profiles. The absence of the change in direction could be due to the number of nodes close to the boundary not being sufficient to model this change (the change is only over three nodes) or it could be a

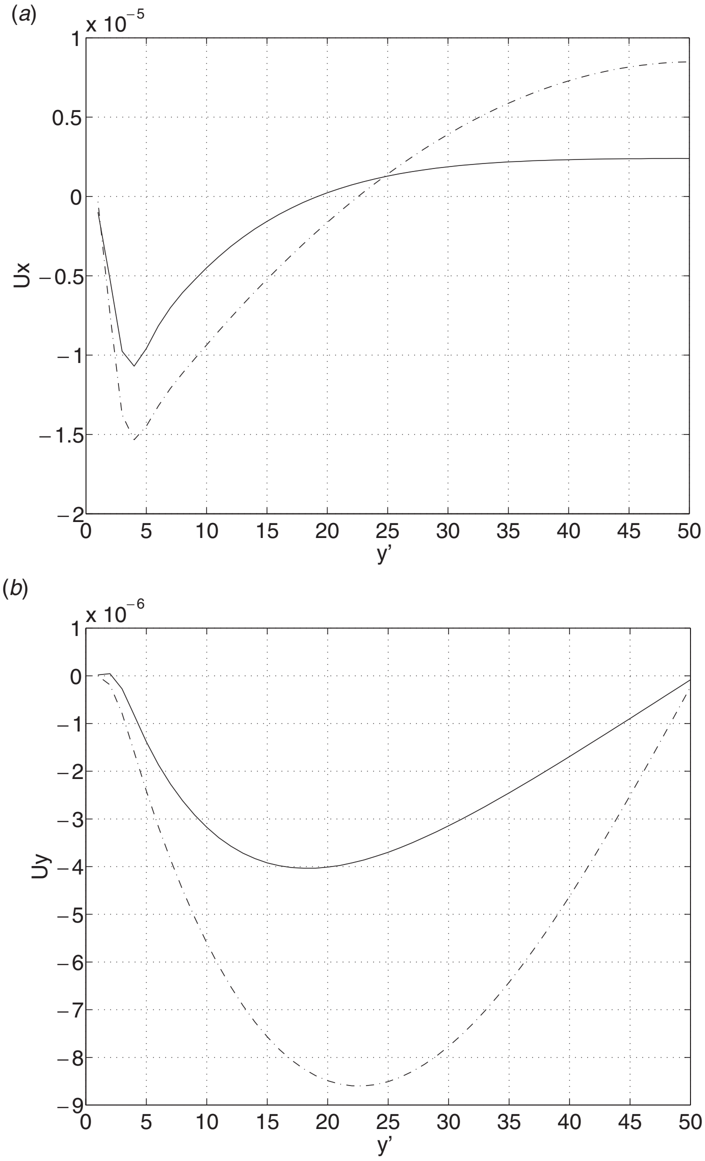
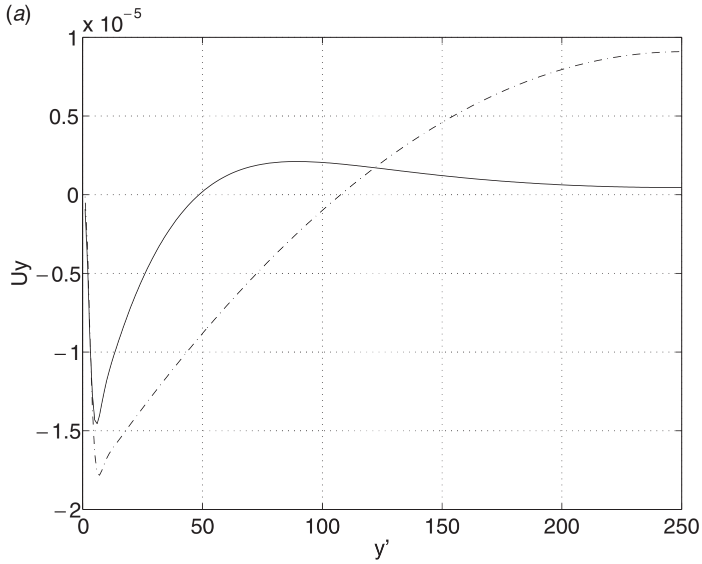


Figure 4. Acoustic streaming at the velocity antinode: (*a*) x-component; (*b*) y-component ($L_x = 200$, $L_y = 86.6$, $\tau = 0.525$, $\triangle\rho = 0.01$, $y' = 2y/\sqrt{3}$). The solid curve is the simulation; the theory is the dashed curve.

non-physical artefact of the mathematical solution. The most likely explanation is the former, as the velocity reversal is seen in the lattice gas simulation [7], though it is greatly exaggerated.

As the width of the channel approaches the wavelength of the standing wave the theoretical approximations will break down and a numerical result is needed. Figures 4 and 5 shows the acoustic streaming velocities for ($L_x = 200$, $L_y = 86.6$) and ($L_x = 540$, $L_y = 433$) respectively. They are reduced, both in the boundary layer and the centre of the channel, compared with the analytical predictions. If the theoretical analysis is used to analyse the



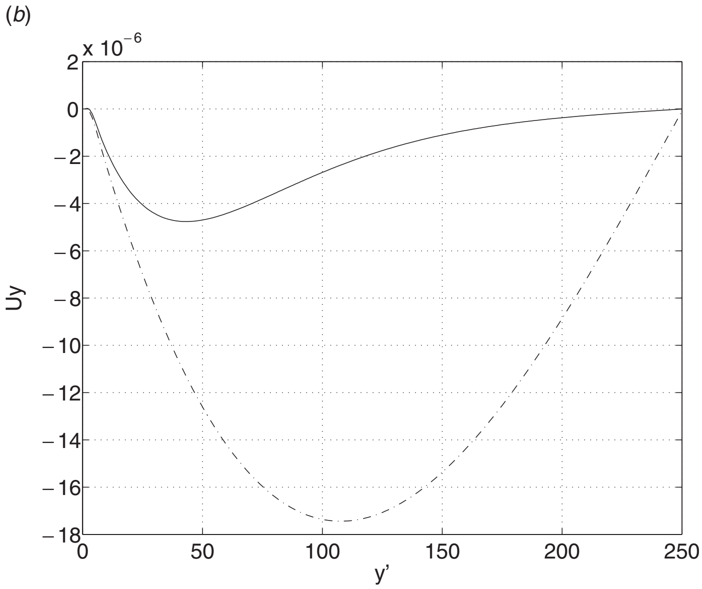


Figure 5. Acoustic streaming at the velocity antinode: (*a*) x-component, (*b*) y-component ($L_x = 540$, $L_y = 433$, $\tau = 0.525$, $\triangle\rho = 0.01$, $y' = 2y/\sqrt{3}$). The solid curve is the simulation; the theory is the dashed curve.

effect of acoustic streaming on rate processes such as diffusion or heat transfer it is likely to overpredict the convective effects as $L_x/L_y \to 1$.

The acoustic streaming profiles produced in the model take a finite time to become established. For a standing wave amplitude of $\triangle\rho_1 = 0.01$ it takes $\sim$250 cycles before the streaming is fully established. If the amplitude is reduced to $\triangle\rho_1 = 0.001$, more than 500 cycles are required. Streaming simulations are also feasible for $\triangle\rho_1 = 10^{-4}$ although the time required to establish streaming is again increased.

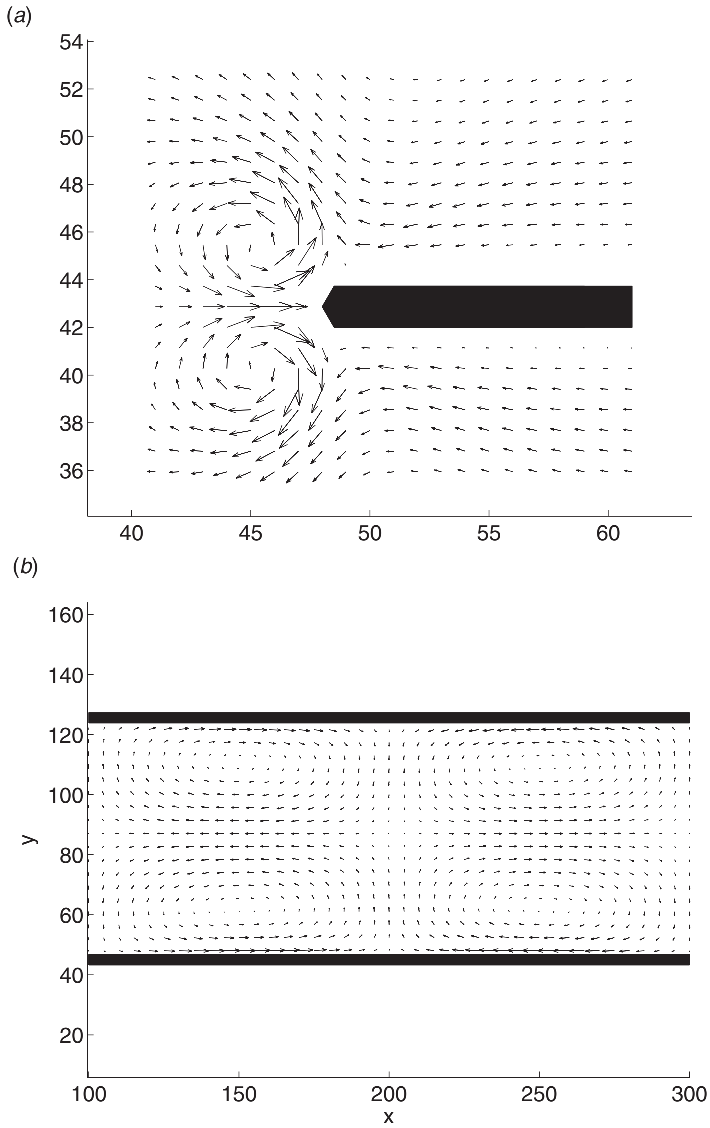


Figure 6. Velocity vector plot of acoustic streaming produced by two thin plates of length 300 lattice points $50 < x < 350$, $y = 43.3$ and $y = 129.9$ on a lattice with $L_x = 400$, $L_y = 173.2$ ($\tau = 0.525$, $\triangle\rho = 0.01$). (*a*) Near the end of a plate; (*b*) in the centre of the plates.

5. Streaming around obstacles

The previous sections have shown that lattice Boltzmann simulations can be used to accurately model acoustic streaming between two parallel plates. This type of streaming is caused by the change in the amplitude of the wave in the boundary layer when it comes into contact with a boundary. Therefore the approach can be extended to modelling the streaming pattern round obstacles in a sound field with any shape that can be reproduced by the lattice. Two examples are presented. The first is the streaming pattern round two thin parallel plates of finite length.

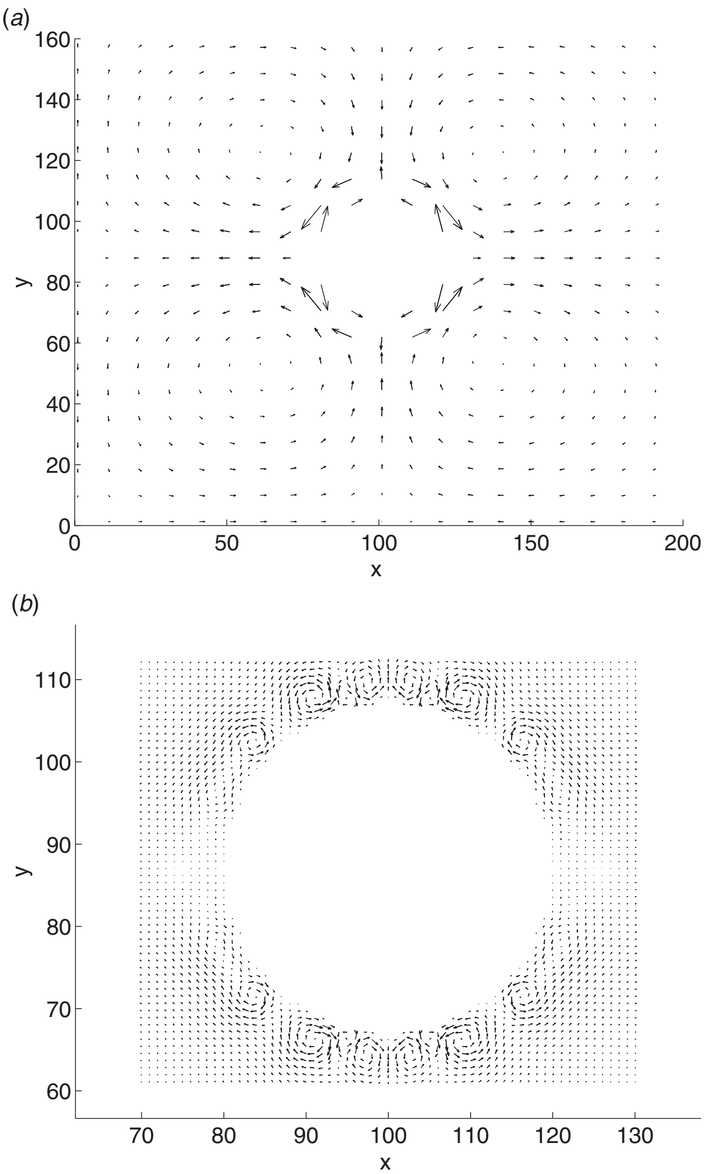


Figure 7. Velocity vector plot of acoustic streaming produced round a 20-lattice-point diameter cylinder on a hexagonal mesh: (*a*) far field; (*b*) flow close to the cylinder ($L_x = 400$, $L_y = 173.2$, $\tau = 0.525$, $\triangle\rho = 0.01$).

The second is the streaming pattern round a cylinder where an analytical result exists [3]. In both simulations the same standing wave field was used as before. The lattice was taken to be periodic in the y-direction and the boundaries of the obstacles were modelled as non-slip using the bounce-back scheme.

Figure 6 shows the streaming profile produced between two parallel thin plates in a standing wave sound field. For this simulation $L_x = 400$, $L_y = 173.2$ and the plates lie at $50 < x < 350$, $y = 43.3$ and $y = 129.9$. To simplify the model the ends of the plate were pointed following the profile of the lattice, allowing a smooth entry into the channel. As expected the streaming profile between the plates is essentially the same as for the infinitely long plates considered in section 3. However, at the entrance the angle of the plates produces rotational streaming patterns similar to those seen round cylinders or round a hexagon for thicker plates.

Figure 7 shows the streaming profile round a cylinder of radius 20 lattice points. Away from the boundary the simulation correctly predicts the four outer vortices as described by Nyborg [3]. However, close to the boundary there are a large number of vortices which result from the sharp edges produced by the lattice approximation to the smooth cylinder. To correct this the boundary conditions need to be modified to more accurately follow the shape of the cylinder [12].

6. Conclusions

In this paper we have shown that lattice Boltzmann simulations can be used to model acoustic streaming which arises from the interaction between a sound wave and a stationary boundary. Like the lattice gas approach, the lattice Boltzmann approach simulates the full Navier–Stokes equations. Therefore once the correct boundary conditions are implemented and a sound wave is imposed streaming is generated automatically. However, using the lattice Boltzmann rather than a lattice gas approach it is possible to produce simulations which are much closer to the theoretical limits. To demonstrate this we have simulated the acoustic streaming produced by a standing wave confined between infinite plates and compared the results to an analytical solution presented by Nyborg.

We have shown that as the relative dimensions of the channel move away from the theoretical limit to a channel width close to the wavelength the streaming velocities both inside and outside the acoustic boundary layer are significantly lower than that predicted by theory, the velocity outside the boundary layer having the greater reduction. This leads to a streaming pattern where the centres of the vortices are closer to the boundary than predicted by the theory. Combined with the overall reduction in velocity this is likely to have a significant impact on predicting the effect of streaming on processes such as diffusion.

Finally we have shown that lattice Boltzmann simulations can be used to simulate streaming round any obstacle which can be represented on the lattice. This was demonstrated by simulating the streaming produced by two plates of finite length (not equal to the wavelength) and a lattice approximation to an infinitely long cylinder.

Having established that lattice Boltzmann simulations can accurately predict Rayleigh streaming it would be interesting to extend the method to include a second species to examine the effect of streaming on diffusion rates in an attempt to understand the physics behind reports of steady enhanced diffusion. Another interesting direction is to examine the streaming produced by the interaction of the sound wave with a bubble suspended in the sound field. It has been suggested that bubbles can produce large streaming velocities, which can damage cellular tissue [13].

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